

LINEAR SCALING WITH CLUSTER SIZE FOR CALCULATIONS OF NMR PROPERTIES IN LIQUIDS

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Dedicated to Professor Rudolf Zahradník on the occasion of his 75th birthday.

It is shown for liquid water that the contributions of the solvent molecules to the NMR chemical shielding of a proton in the solute molecule is approximately pair-additive. This leads to linear scaling in the computational effort with cluster size if the cluster method is applied. Alternatively, a pair chemical shielding surface, in analogy to the pair potential surface, can be calculated in advance of the simulation. Its analytical fit can be coded into the simulation program to be easily evaluated any time during the simulation, *e.g.* to get time correlation functions. As water is an extreme case, it is assumed that for other systems an even better additivity might be found.

Keywords: NMR chemical shielding; Pair-additivity; Liquid state; Cluster method; Linear scaling; Molecular dynamics; DQCC.

One of the most promising methods for the calculation of NMR properties in liquids is the cluster method^{1,2}. In this method clusters around an atom of interest, *e.g.* a hydrogen, are extracted from configurations obtained in MC or MD simulations. They are used in supermolecular quantum-chemical calculations to obtain the property of interest, *e.g.* the proton chemical shift in a solvated molecule. Repeating this procedure with many simulation-snapshots and averaging the supermolecular property, the property in solution for the given phase point is obtained. The cluster size has to be chosen large enough that interactions from molecules further away can be neglected. As the computing time with most quantum chemical methods is growing with powers between three and six and typically hundreds of clusters are needed to yield an average for the liquid, the cluster size and, hence, convergence to the bulk property, becomes a limiting factor. However, for the deuteron quadrupole coupling constant (DQCC) in water and in a mixture of water and DMSO, we found that the DQCC are highly pair-

additive³. The striking additivity is best seen in a graph, where the components of the DQCC tensor obtained in a pair-additive way are plotted against the ones obtained in a supermolecular calculation for the whole cluster. This is shown for 600 components (the three diagonal and three off-diagonal elements of tensors obtained for 100 clusters) of a water/DMSO mixture in Fig. 1 (a similar plot for neat water is shown in Fig. 1 of ref.³).

In recent years Farrar and coworkers⁴⁻⁶ found empirically a linear relationship between chemical shieldings of protons and the corresponding DQCCs. This together with the above pair-additivity led us to the conclusion that there should also exist an approximate linear relation between chemical shieldings for a proton in a molecule in the centre of a cluster obtained once by a calculation of the whole cluster and once by adding all pair-interactions of the outer molecules to the shielding in the central one, *i.e.* using the following equation:

$$\sigma_i = \sigma_i^{\text{monomer}} + \sum_{j \neq i} \sigma_{ij}^{\text{dimer}}, \quad (1)$$

where σ_i is the shielding in the solute molecule i in the liquid, $\sigma_i^{\text{monomer}}$ the shielding in the isolated solute molecule i and $\sigma_{ij}^{\text{dimer}}$ is defined as $\sigma_{ij}^{\text{dimer}} = \sigma_{ij} - \sigma_i^{\text{monomer}}$ with σ_{ij} being the shielding in the solute molecule in the pair consisting of the solute molecule i and a solvent molecule j . We checked this for the proton shielding with the same 100 water clusters used

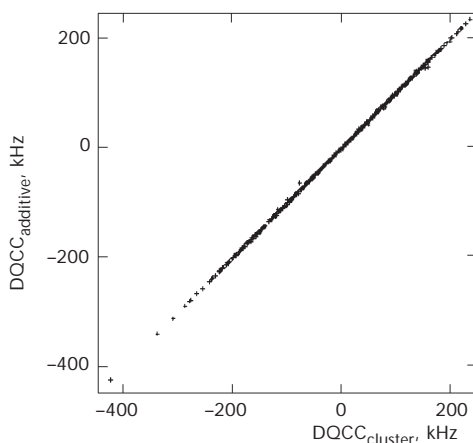


FIG. 1

600 components of the deuteron quadrupole coupling tensors in D_2O for 100 clusters from a water/DMSO mixture with a mole fraction $x_{\text{water}} = 0.3$

before for the DQCC calculations³. All technical details like basis set, method (Hartree–Fock) and cluster size (14 to 22 water molecules) were the same as before³. The shielding calculations were performed by the GIAO method^{7,8}. The results in Fig. 2 show that this linearity exists, although with a lower quality than for the DQCCs.

The RMS deviation between the shieldings (in ppm) obtained from the cluster calculation and the one assuming pair-additivity is 0.5 ppm. A linear regression (see the line in Fig. 2) yields

$$\sigma_{\text{additive}} = (0.961 \pm 0.008)\sigma_{\text{cluster}} + (1.57 \pm 0.022) \quad (2)$$

with a correlation coefficient of 0.997 and a standard deviation $s(\sigma_{\text{additive}}) = 0.18$ ppm.

If this finding is used for the calculation of absolute proton shieldings of a size of about 25 ppm, the accuracy is excellent. However, applying the cluster method, one is mostly interested in the gas–liquid shift of an NMR property. In the present case a value of (-3.15 ± 0.18) ppm is calculated from the clusters (experimental value -4.3 ppm; for other calculated values, see ref.²). Although the RMS error due to the assumption of pair-additivity of 0.5 ppm is relatively large (16% of 3.15 ppm), it is still smaller than the deviation from experiment. An improvement would be to calculate a few clusters first to obtain the linear relationship and then to calculate σ_{cluster} from σ_{additive} by this relation for the remaining clusters, hence reducing the error roughly to the standard deviation of 0.18 ppm for predictions from

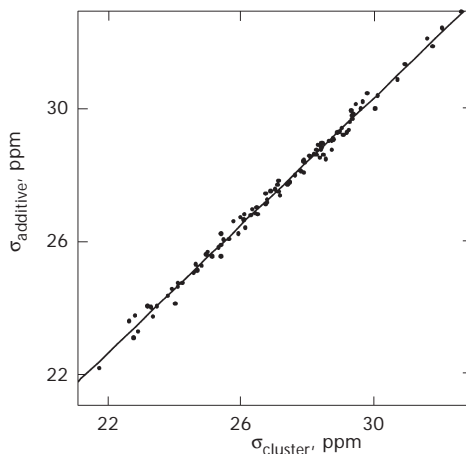


FIG. 2

Isotropic chemical shielding of a proton of the central water molecule in 100 water clusters

the linear regression. This statistical error is probably smaller than the error of any predictive computational method for liquid-gas shifts available in the next years will be.

CONCLUSIONS

Let us summarise what one can gain from the present findings for the calculation of NMR properties of molecules in solution.

The scaling with cluster size in the cluster method becomes linear, which permits use of large clusters, yielding NMR properties of the bulk in the large cluster limit.

To obtain statistically accurate dynamic properties, like relaxation times, which are calculated by integration of time correlation functions, one would need a prohibitive number of quantum chemical calculations. Therefore, other methods are needed to calculate these properties. A feasible way is to assume pair-additivity for the property and to calculate a property surface for a pair of molecules in advance of the simulation completely analogous to the potential surface. An analytical fit to the surface points coded into the simulation program is then easily evaluated in each step of the simulation. This procedure has been used to obtain the quadrupolar relaxation time in simple systems like spherical ions in water⁹ and for liquid neon¹⁰⁻¹². After the demonstration of additivity for DQCCs³, we have applied it for the first time to more complicated systems like water¹³ and water/DMSO mixtures¹⁴.

Further work is needed to demonstrate pair-additivity or linear relationships for other chemical systems, other nuclei and other properties. We are very confident, however, that such relationships are valid in less polar systems and systems without hydrogen bonds, as we have shown that they are even valid for the extreme case of water.

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